PAST EXAM PAPER & MEMO N3

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This question paper consists of 6 pages and 1 formula sheet of 2 pages.
DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA
NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.

2. Read ALL the questions carefully.

3. Number the answers according to the numbering system used in this question paper.

4. Questions may be answered in any order but subsections of questions must NOT be separated.

5. Show ALL the calculations and intermediary steps.

6. ALL final answers must be accurately approximated to THREE decimal places.

7. ALL graph work must be done in the ANSWER BOOK. Graph paper is NOT supplied.

8. Diagrams are NOT drawn to scale.

9. Write neatly and legibly.
QUESTION 1

1.1 Factorise the following expressions as far as possible in prime factors:

1.1.1 \( x(3x - 2) - y(3y + 2) \) \( \text{(4)} \)

1.1.2 \( 4n^4p + 3n^2p - 1 \) \( \text{(2)} \)

1.2 Factorise the following expression completely:

\( 2x^3 + x^2 - 5x + 2 \) \( \text{(5)} \)

1.3 Simplify the following expression:

\[
\frac{x - 1}{x + 1} - \frac{2x - 1}{3 - x} + \frac{2x^2 - 7x - 17}{x^2 - 2x - 3}
\]

\( \text{(6)} \)

QUESTION 2

2.1 Simplify the following:

\[
\sqrt{x} - \frac{1}{2\sqrt{x}}
\]

\[
\frac{3}{x^2}
\]

\( \text{(4)} \)

2.2 Use logs to the base 2 and simplify the following WITHOUT using a calculator:

\( \log_{0.5} 128 \) \( \text{(4)} \)

2.3 Solve for \( x \):

2.3.1 \( \sqrt{16 + 3^x} = 3^x + 2 \) \( \text{(2 \times 4)} \)

2.3.2 \( (\log x - 2) \times \log(x - 2) = 0 \) \( \text{(8)} \)
QUESTION 3

3.1 Solve for \( x \) by completing the square:
\[ 4x = 48 - 2x^2 \]

3.2 Make ‘\( b \)’ the subject of the formula:
\[ D = \frac{x + b}{\sqrt{x - b}} \]

3.3 Make ‘\( w \)’ the subject of the formula:
\[ \log_e t - \log_e p + \log_e w = ds \]

3.4 Alex paid a deposit of \( R3x \) for a computer. He paid the rest in 9 monthly instalments. He paid a total of \( R33x \). What is the payment of each monthly instalment in terms of \( x \).

QUESTION 4

4.1 Consider FIGURE A below. \( \triangle ABC \) is an isosceles triangle with \( AB = BC \) and vertices \( A(2;1), B(4;5) \) and \( C(0;k) \).
4.1.1 Find the length of AB.  

4.1.2 Determine the value(s) of \( k \).  

4.1.3 Show that \( AB \) is perpendicular to \( BC \) if \( k = 7 \).  

4.1.4 Calculate the area of \( \triangle ABC \) when \( k = 7 \).  

4.2 \( P(-2;-1) \) and \( Q(4;7) \) are points in the plane with \( M \) as the midpoint of \( PQ \). Determine the equation of the line parallel to the \( y \)-axis and passing through the point \( M \).  

4.3 Consider FIGURE B below. The lines \( BA \) and \( CA \) with equations \( y = x + 2 \) and \( y = 3x - 3 \) respectively, intersect at \( A \). Determine the size of \( \angle BAC \) where \( B \) and \( C \) are the intercepts on the \( x \)-axis as shown.
QUESTION 5

5.1 Draw the graph defined by the equation: \(3x^2 + 3y^2 = 27\)

5.2 Given: \(y = x^3 + 6x^2 + 9x\)

5.2.1 Make use of differentiation to determine the coordinates of the turning points of the given equation.

5.2.2 Draw the graph of the given function. Show ALL values at the points of intersection with the system of axes and the co-ordinates of the turning points.

5.3 Determine \(\frac{dy}{dx}\) if \(y = \frac{1}{x} + 2\sqrt{x}\). Leave the answers with positive indices and in surd form.

QUESTION 6

6.1 Prove the following trigonometric identity:

\[\sin^2 A + \tan^2 A + \cos^2 A = \sec^2 A\]

6.2 Calculate the value(s) of \(\theta\) which will satisfy the equation if \(0^\circ \leq \theta \leq 270^\circ\):

\[\sin \theta = 1 - \cos^2 \theta\]

6.3 Consider FIGURE C below. An observer, standing at a point A is watching the top of a vertical tower BC. The angle of elevation of the top of the tower, BC, is 25° and the angle of depression of the foot of the tower is 20°. If the height of the tower BC is known to be 30 m, determine the following:

6.3.1 The distance between the observer at A and the point B.

6.3.2 The distance between the two towers.
6.4 Consider FIGURE D below. The sketch represents the graph of \( f(x) = a \sin px \) where \( 0 \leq x \leq \pi \). Determine the values of \( a \) and \( p \). 

FIGURE D
FORMULA SHEET

Any applicable formula may also be used.

1. Factors

\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]
\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]

2. Logarithms

\[ \log ab = \log a + \log b \]
\[ \log \frac{a}{b} = \log a - \log b \]
\[ \log_b a = \frac{\log_c a}{\log_c b} \]
\[ \log a^m = m \log a \]
\[ \log_b a = \frac{1}{\log_a b} \]
\[ \log_a a = 1 \quad \ln e = 1 \]
\[ a^{\log_a t} = t \quad e^{\ln m} = m \]

3. Quadratic formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

4. Parabola

\[ y = ax^2 + bx + c \]
\[ y = 4ac - b^2 \]
\[ x = \frac{-b}{2a} \]

5. Circle

\[ x^2 + y^2 = r^2 \]
\[ D = \frac{x^2}{4h} + h \]
\[ x = \sqrt{4Dh - 4h^2} \]

6. Straight line

\[ y - y_1 = m(x - x_1) \]

Perpendicular: \( m_1 \cdot m_2 = -1 \)

Parallel lines: \( m_1 = m_2 \)

Distance: \( D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

Midpoint: \( P = \left( \frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right) \)

Angle of inclination: \( \theta = \tan^{-1} m \)
7. Differentiation

\[
\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
\frac{d}{dx} (x^n) = nx^{n-1}
\]

Max/Min

For turning points: \( f'(x) = 0 \)

8. Trigonometry

\[
\sin \theta = \frac{y}{r} = \frac{1}{\csc \theta}
\]

\[
\cos \theta = \frac{x}{r} = \frac{1}{\sec \theta}
\]

\[
\tan \theta = \frac{y}{x} = \frac{1}{\cot \theta}
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

\[
1 + \tan^2 \theta = \sec^2 \theta
\]

\[
1 + \cot^2 \theta = \csc^2 \theta
\]

\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]

\[
\cot \theta = \frac{\cos \theta}{\sin \theta}
\]

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

Area of \( \triangle ABC = \frac{1}{2} ac \sin B \)
MARKING GUIDELINE

NATIONAL CERTIFICATE

APRIL EXAMINATION

MATHEMATICS N3

1 APRIL 2016

This marking guideline consists of 10 pages.
QUESTION 1

1.1 1.1.1 \[ x(3x - 2) - y(3y + 2) \]
\[ = 3x^2 - 2x - 3y^2 - 2y \quad \checkmark \]
\[ = 3(x^2 - y^2) - 2(x + y) \quad \checkmark \]
\[ = 3(x - y)(x + y) - 2(x + y) \]
\[ = (x + y)[3(x - y) - 2] \]
\[ = (x + y)[3x - 3y - 2] \quad \checkmark \quad \checkmark \quad (4) \]

1.1.2 \[ 4n^{3p} + 3n^{2p} - 1 \]
\[ = (n^{2p} + 1)(4n^{3p} - 1) \quad \checkmark \quad \checkmark \quad (2) \]

1.2 \[ f(x) = 2x^3 + x^2 - 5x + 2 \]
\[ \therefore f(1) = 2(1)^3 + (1)^2 - 5(1) + 2 \]
\[ = 2 + 1 - 5 + 2 \quad \checkmark \]
\[ = 0 \quad \checkmark \]
\[ \therefore x - 1 \text{ is a factor of } f(x) \quad \checkmark \]
\[ x - 1 \]
\[ \begin{array}{c|c}
2x^2 + 3x - 2 \\
\hline
2x^3 + x^2 - 5x + 2 \\
2x^3 - 2x^2 \\
3x^2 - 5x \\
3x^2 - 3x \\
-2x + 2 \\
-2x + 2 \\
\end{array} \]
\[ \therefore f(x) = (x - 1)(2x^2 + 3x - 2) \]
\[ = (x - 1)(2x - 1)(x + 2) \quad \checkmark \quad \checkmark \quad (5) \]
1.3 \[ \frac{x - 1}{x + 1} - \frac{2x - 1}{3 - x} + \frac{2x^2 - 7x - 17}{x^2 - 2x - 3} \]

\[ = \frac{x-1}{x+1} + \frac{2x-1}{x-3} + \frac{2x^2-7x-17}{(x+1)(x-3)} \] (\( \checkmark \checkmark \))

\[ = \frac{(x-1)(x-3) + (2x-1)(x+1) + 2x^2 - 7x - 17}{(x+1)(x-3)} \] (\( \checkmark \))

\[ = \frac{x^2 - 4x + 3 + 2x^2 + x - 1 + 2x^2 - 7x - 17}{(x+1)(x-3)} \] (\( \checkmark \))

\[ = \frac{5x^2 - 10x - 15}{(x+1)(x-3)} \]

\[ = \frac{5(x^2 - 2x - 3)}{(x+1)(x-3)} \] (\( \checkmark \))

\[ = \frac{5(x+1)(x-3)}{(x+1)(x-3)} \]

\[ = 5 \] (6) [17]
QUESTION 2

2.1 \[ \sqrt{x} - \frac{1}{2\sqrt{x}} \]
\[ = \frac{x}{3} + 3x^2 \]
\[ = \frac{2x-1}{1} \times \frac{1}{3} \]
\[ = \frac{2x-1}{6x^2} \]
\[ = \frac{2x-1}{6x^2} \] (4)

2.2 \[ \log_{0.5} 128 \]
\[ = \frac{\log 128}{\log 2} \]
\[ = \log 2^{7} \]
\[ = \frac{7 \log 2}{-1 \log 2} \]
\[ = -7 \] (4)

2.3 2.3.1 \[ \sqrt{16 + 3^x} = 3^x + 2 \]
\[ \therefore 16 + 3^x = (3^x + 2)^2 \] (4)
\[ \therefore 16 + 3^x = 3^{2x} + 4.3^x + 4 \] (4)
\[ \therefore 12 = 4.3^x \]
\[ \therefore 3^x = 3 \] (4)
\[ \therefore x = 1 \] (4)
TEST : If \( x = 1 \) then LHS=RHS=5

2.3.2 \[ (\log x - 2) \times \log(x - 2) = 0 \]
\[ \therefore \log x - 2 = 0 \text{ or } \log(x - 2) = 0 \]
\[ \therefore \log x = 2 \]
\[ \therefore x = 10^2 \]
\[ \therefore x = 100 \]
\[ \therefore x = 3 \] (4)

[16]
QUESTION 3

3.1 \[ 4x = 48 - 2x^2 \]

\[ 2x^2 + 4x = 48 \checkmark \]
\[ x^2 + 2x = 24 \]
\[ x^2 + 2x + 1 = 24 + 1 \checkmark \]
\[ (x + 1)^2 = 25 \]
\[ x + 1 = \pm \sqrt{25} \checkmark \]
\[ x = -1 \pm 5 \]
\[ x = -6 \text{ or } x = 4 \checkmark \]

\[ 3.2 \quad D = \frac{x + b}{x - b} \]
\[ D^2 = \frac{x + b}{x - b} \checkmark \]
\[ D^2x - D^2b = x + b \checkmark \]
\[ D^2b + b = D^2x - x \]
\[ b(D^2 + 1) = D^2x - x \checkmark \]
\[ b = \frac{D^2x - x}{D^2 + 1} = \frac{(D^2 - 1)x}{D^2 + 1} \checkmark \]
3.3 \[ \log_s t - \log_s p + \log_s w = ds \]

\[ \log_s \frac{tw}{p} = ds \checkmark \]

\[ \frac{tw}{p} = e^{ds} \checkmark \]

\[ w = \frac{p}{t} e^{ds} \checkmark \]

(3)

3.4 Deposit is R3x \checkmark 
Total price is R33x \checkmark 
Total amount for 9 instalments= R30x \checkmark 
\[ \therefore \text{ Each monthly instalment } = R \frac{30x}{9} = R \frac{10x}{3} \checkmark \checkmark \]

(4)

[15]
QUESTION 4

4.1 4.1.1  \[ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(4 - 2)^2 + (5 - 1)^2} \]
\[ = \sqrt{4 + 16} \quad \checkmark \]
\[ = \sqrt{20} \quad \checkmark \]
\[ = 2\sqrt{5} \quad \checkmark \]  
\[ \text{(2)} \]

4.1.2  \[ BC = 2\sqrt{5} \]
\[ \therefore \sqrt{(0 - 4)^2 + (k - 5)^2} = \sqrt{20} \]
\[ \therefore 16 + (k - 5)^2 = 20 \quad \checkmark \]
\[ \therefore (k - 5)^2 = 4 \]
\[ \therefore k - 5 = \pm 2 \quad \checkmark \]
\[ \therefore k - 5 = 2 \quad \text{or} \quad k - 5 = -2 \]
\[ \therefore k = 7 \quad \checkmark \quad k = 3 \quad \checkmark \]  
\[ \text{(4)} \]

4.1.3  \[ \therefore M_{AB} = \frac{5 - 1}{4 - 2} = 2 \quad \checkmark \]
\[ \therefore M_{CB} = \frac{7 - 5}{0 - 4} = -\frac{1}{2} \quad \checkmark \]
\[ M_{AB} \times M_{CB} = -1 \quad \checkmark \]
\[ \text{therefore} \quad CB \perp AB \quad \checkmark \]  
\[ \text{(3)} \]

4.1.4  \[ \text{Area of } \triangle ABC = \frac{1}{2} \times \text{base} \times \text{height} \]
\[ = \frac{1}{2} AB \times BC \quad \checkmark \]
\[ = \frac{1}{2} \sqrt{20} \times \sqrt{20} \quad \checkmark \]
\[ = 10 \text{ units}^2 \quad \checkmark \]  
\[ \text{(3)} \]
4.2

The coordinates of M are given by

\[ M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

\[ = \left( \frac{-2 + 4}{2}, \frac{-1 + 7}{2} \right) \]

\[ = (1;3) \]

\[ \therefore \text{Equation } x=1 \]

\[ \left(4\right) \]

4.3

\[ M_{AB} = 1 \]

\[ M_{AC} = 3 \]

\[ \therefore \tan \theta = 1 \]

\[ \therefore \tan \alpha = 3 \]

\[ \therefore \theta = 45^\circ \]

\[ \therefore \alpha = 71.565^\circ \]

\[ B\hat{A}C = \alpha - \theta = 71.565^\circ - 45^\circ = 26.565^\circ \]

\[ \left(5\right) \]

5.1

\[ 3x^2 + 3y^2 = 27 \]
5.2.1 \[ y = x^3 + 6x^2 + 9x \]
\[ \frac{dy}{dx} = 3x^2 + 12x + 9 \quad \checkmark \]
Let \( y = 0 \)
\[ \therefore +3x^2 + 12x + 9 = 0 \]
\[ 3(x^2 + 4x + 3) = 0 \]
\[ (x+1)(x+3) = 0 \quad \checkmark \quad \checkmark \]
\[ x = -1 \quad \text{or} \quad x = -3 \]
\[ f(-1) = -(-1)^3 - 6(-1)^2 - 9(-1) = 4 \]
\[ f(-3) = -(-3)^3 - 6(-3)^2 - 9(-3) = 0 \]
Turning points \((-3;0)\) and \((-1;4)\)

5.2.2

5.3
\[ y = \frac{1}{x} + 2\sqrt{x} \]
\[ y = x^{-1} + 2x^{\frac{1}{2}} \quad \checkmark \]
\[ \frac{dy}{dx} = -x^{-2} + 2(0.5)x^{-\frac{1}{2}} \quad \checkmark \]
\[ \frac{dy}{dx} = -\frac{1}{x^2} + \frac{1}{\sqrt{x}} \quad \checkmark \quad \checkmark \]
QUESTION 6

6.1 \[ LHS = \sin^2 A + \tan^2 A + \cos^2 A \]
   \[ = (\sin^2 A + \cos^2 A) + \tan^2 A \checkmark \]
   \[ = 1 + \tan^2 A \checkmark \checkmark \]
   \[ = \sec^2 A = RHS \checkmark \] (4)

6.2 \[ \sin \theta = 1 - \cos^2 \theta \]
   \[ \therefore \sin \theta = \sin^2 \theta \checkmark \]
   \[ \therefore \sin^2 \theta - \sin \theta = 0 \checkmark \]
   \[ \therefore \sin \theta (\sin \theta - 1) = 0 \checkmark \]
   \[ \therefore \theta = 0^\circ \text{ or } \theta = 90^\circ \checkmark \]
   \[ \text{or } \theta = 180^\circ \checkmark \checkmark \] (5)

6.3 6.3.1

\[ \text{In } \triangle ABC: \]
\[ \frac{AB}{\sin 70^\circ} = \frac{30}{\sin 45^\circ} \checkmark \]
\[ \therefore AB = \frac{30 \sin 70^\circ}{\sin 45^\circ} \checkmark \]
\[ = 39,868 m \] (3)

6.3.2

\[ \text{In } \triangle ABD: \]
\[ \frac{AD}{AB} = \cos 25^\circ \checkmark \]
\[ \therefore AD = AB \cos 25^\circ \checkmark \]
\[ = 39,868 \times \cos 25^\circ \]
\[ = 36,133 m \checkmark \] (3)
6.4 \[ a = 2 \] ✓
\[ p = 2 \] ✓

(2)

[17]

TOTAL: 100
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